or, $m\frac{d^2x}{dt^2} = -Sx - \mu\frac{dx}{dt} + f_0 \sin pt$

or, $\frac{d^2x}{dt^2} + \frac{\mu}{m}\frac{dx}{dt} + \frac{S}{m}x = \frac{f_0}{m}\sin pt$

Putting $\frac{\mu}{m} = 2K$, $\frac{S}{m} = \omega^2$ and $\frac{f_0}{m} = F_0$, we get -

$$\boxed{\frac{d^2x}{dt^2} + 2K\frac{dx}{dt} + \omega^2 x = F_0 \sin pt} \longrightarrow (2)$$

This is the differential equation of motion of the forced harmonic oscillator.

Once the transient effect dies away and a steady state is set up the system is oscillate with the frequency of the applied force. This is the necessary physical condition of the oscillations.

Now let us suppose the particular solution of this equation is,

$$x = A\sin(pt - \theta) \longrightarrow (3)$$

$$\therefore \frac{dx}{dt} = Ap\cos(pt-\theta)$$

and $\frac{d^2x}{dt^2} = -Ap^2\sin(pt-\theta)$

Substituting these value in equation (2) we get -

$$(-p^2A + \omega^2A)\sin(pt-\theta) + 2KpA\cos(pt-\theta) = F_0\sin(pt-\theta)\cos\theta + F_0\cos(pt-\theta)\sin\theta$$

This is an identity and if this equation is to be satisfied for all values of t, then the coefficients of $\sin(pt-\theta)$ and $\cos(pt-\theta)$ on the two sides of this equation must be equal after solving of above equation and substituting these values of A and θ in equation (3) we get -

$$x = \frac{F_0}{\sqrt{\{(\omega^2-p^2)^2 + 4K^2p^2\}}}\sin\left\{pt - \tan^{-1}\left(\frac{2Kp}{\omega^2-p^2}\right)\right\} \longrightarrow (4)$$

This is the solution of the differential equation of the forced harmonic oscillator.